INDIAN MARITIME UNIVERSITY

(A CENTRAL UNIVERSITY, GOVT. OF INDIA)

December 2017 End Semester Examination
B. Tech. (Marine Engineering) First Semester

Mathematics - I (Subject Code: UG11T3102/ UG11T2102 UG11T1102)

Date: 07/12/2017 **Max Marks:** 100
Time: 3 Hrs. **Pass Marks:** 50

PART – A **(3 x10 = 30)**

Compulsory Questions: (The symbols have their usual meanings.)

1. (a) Find the nth derivative of $\log(4x^2-1)$.

(b) If $z=e^{ax+by}f(ax-by)$, prove that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=2abz$.

(c) Find the radius of curvature at any point $(at^2, 2at)$ of the parabola $y^2=4ax$.

(d) If $u=xy-yz-zx, v=x^2+y^2+z^2$ and $w=x+y-z$, determine whether they are functionally related, if so find the relationship between them.

(e) If $u=e^{xy}$, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{u}\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right\}$.

(f) Evaluate the integral $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing to polar coordinates.

(g) Show that $\int_0^\infty \frac{x^4}{4^x}dx=\frac{\Gamma(5)}{(\log 4)^5}$.

(h) Find the directional derivative of function $f=xy^2+yz^2$ at the point $(1,-1,1)$ along the vector $\hat{\imath}+2\hat{\jmath}+\hat{k}$.

(i) Using Cayley Hamilton theorem find the A^{-1} of matrix $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

(j) Evaluate the integral $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$, where C is the circle $|z|=1/2$.

PART – B **(14 x 5 = 70)**

Answer any <u>FIVE</u> of the following questions

2. (a) If $y = e^{m\cos^{-1}x}$, prove that
   (i) $(1-x^2)y_2 - xy_1 = m^2y$
   (ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. **[3+4]**

   (b) Find the asymptotes of the curve $y^3 - 2xy^2 - x^2y + 2x^3 + 3y^2 - 7xy + 2x^2 + 2y + 2x + 1 = 0$. **[7]**

3. (a) If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, prove that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$. **[7]**

   (b) Using the method of Lagrange's multipliers find the points on the surface $z^2 = xy + 1$ nearest to the origin. **[7]**

4. (a) Using the rule of differentiation under the sign of integration prove that $\int_0^{\pi/2}\frac{\log(1+y\sin^2 x)}{\sin^2 x}dx = \pi\left[\sqrt{(1+y)} - 1\right]$. **[7]**

   (b) Prove that
   (i) $\beta\left(m,\frac{1}{2}\right) = 2^{2m-1}\beta(m,m)$ and hence
   (ii) $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. **[5+2]**

5. (a) Evaluate the double integral $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$. **[7]**

   (b) Evaluate triple integral $\int_1^e\int_1^{\log y}\int_1^{e^x}\log z\,dz\,dx\,dy$. **[7]**

6. (a) A particle moves on the curve $x = 2t^2,\ y = t^2 - 4t,\ z = 3t - 5$ where t is the time. Find the component of the velocity and acceleration at $t = 1$ in the direction $i - 3j + 2k$. **[6]**

   (b) Show that vector $\vec{F} = (2xy^2 + yz)\hat{\imath} + (2x^2y + xz + 2yz^2)\hat{\jmath} + (2y^2z + xy)\hat{k}$ is irrotational. Find a scalar function $\emptyset$ such that $\vec{F} = \nabla\emptyset$ **[8]**

7. (a) Discuss the consistency of the following system of equations and solve it if consistent.
   $x + 2y - z = 3$,
   $3x - y + 2z = 1$,
   $2x - 2y + 3z = 2$,
   $x - y + z = -1$ **[7]**

   (b) Let $F(z) = u(x,y) + iv(x,y)$ be an analytic function of z. If $u = x^4 - 6x^2y^2 + y^4$ then find the v and express $f(z)$ in terms of z. **[7]**

8. (a) Find the curve on which functional $\int_0^2 (x + y')y'dx$ with $y(0) = 0$ and $y(2) = 1$ can be extremized. **[7]**

(b) Using simplex method solve the following LPP
Maximize $Z = 5x_1 + 3x_2$
subject to $x_1 + x_2 \leq 2,\ 5x_1 + 2x_2 \leq 10,\ 3x_1 + 8x_2 \leq 12,\ x_1,\ x_2 \geq 0.$ **[7]**

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